Bose–Hubbard Simulators for Gauge Theories

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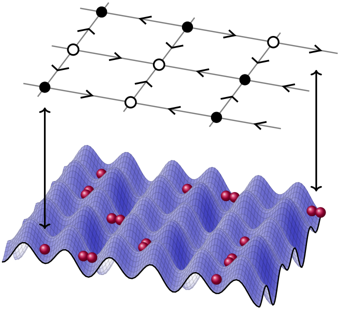
Introduction

- Gauge theories describe particles interacting via a force.
- Can simulate gauge theory dynamics using bosons in an optical lattice.
- Explore unique phenomena, e.g. confinement.



Quantum simulators vs quantum computers

(Digital) quantum computers:

- Programmable qubits.
- Limited by noise.

(Analogue) quantum simulators:

- Specialised to a certain problem.
- Limited by coherence time.

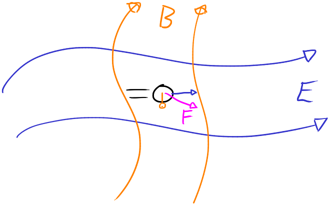
Classical computers (e.g. tensor networks):

- No noise.
- Limited by growth of entanglement.

Outline

- Background: Lattice gauge theories.
- 1+1D simulator with spin-1/2 gauge fields.
- Spin-1 gauge fields.
- 2+1D.

Classical electrodynamics



A charged particle in an electromagnetic field experiences the force

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}).$$

Can obtain the field by Maxwell's equations.

Classical electrodynamics cont.

Can write **E** and **B** in terms of 4-vector potential $A^\mu = (\phi, \mathbf{A})$.

Maxwell's equations become

$$\partial_\mu F^{\mu\nu} = j^\nu,$$

where

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu, \qquad j^\mu = (\rho, \mathbf{j}).$$

Lagrangian:

$$\mathscr{L} = -j^\mu A_\mu - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}.$$

Quantum electrodynamics

Dirac Lagrangian:

$$\mathscr{L} = \bar{\psi}(\mathrm{i}\not{\partial} - m)\psi.$$

Bispinor field, describing electrons and positrons:

$$\psi = \begin{pmatrix} \psi_{\text{electron}} \\ \psi_{\text{positron}} \end{pmatrix}.$$

Combine to form QED:

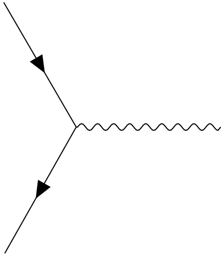
$$\mathscr{L} = \bar{\psi}(\mathrm{i}\not{\partial} - m)\psi - j^{\mu}A_{\mu} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}, \quad \text{where} \quad j^{\mu} = e\bar{\psi}\gamma^{\mu}\psi.$$

QED cont.

In terms of the *gauge covariant derivative* D_μ:

$$\mathscr{L} = \bar{\psi}\left(\mathrm{i}\not{D} - m\right)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}, \qquad D_\mu = \partial_\mu + \mathrm{i}eA_\mu.$$

Key points:



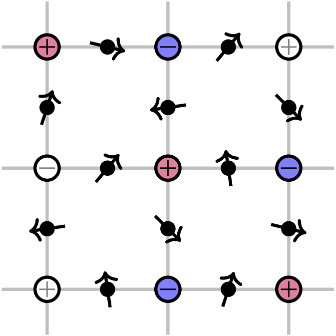
- The dynamics of particles is coupled to the gauge field A^μ (photons).
- ψ and A^μ are invariant under U(1) gauge transformations.

Lattice QED

$$\hat{H}_{\text{QED}} = -\frac{\kappa}{2a} \sum_{\langle i,j \rangle} \left(\hat{\psi}_i^\dagger \hat{U}_{ij} \hat{\psi}_j^\dagger + \text{H.c.} \right) + m \sum_i \hat{\psi}_i^\dagger \hat{\psi}_i$$

$$+ \frac{a}{2} \sum_{\langle i,j \rangle} \left(\hat{E}_{ij} + E_{\text{background}} \right)^2 .$$

- Lattice spacing a.
- Staggered particles:
  even = matter,
  odd = antimatter.
- Gauge sites ($\hat{U}$, $\hat{E}$) on edges.



Quantum link models

$\hat{U}$ and $\hat{E}$ must satisfy (g = gauge coupling strength)

$$[\hat{E}, \hat{U}] = -g\hat{U}, \qquad [\hat{U}, \hat{U}^\dagger] = 0.$$

Approximate using spin-S operators:

$$\hat{U} \to \frac{\hat{S}^-}{\sqrt{S(S+1)}}, \qquad \hat{E} \to g\hat{S}^z.$$

Recover QED in the limit $S \to \infty$.

Gauge invariance

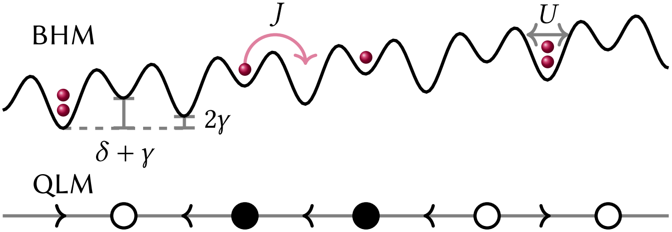
The U(1) gauge symmetry is generated by

$$\hat{G}_i = (-1)^{x_i+y_i}\left[\hat{\psi}_i^\dagger\hat{\psi}_i + \sum_{j \text{ next to } i} \hat{S}_{ij}^z\right]$$

We restrict ourselves to gauge-invariant states satisfying $\langle\hat{G}_i\rangle = 0$.

For a given matter configuration, this restricts the allowed configuration for the gauge sites.

1+1D spin-1/2 bosonic simulator

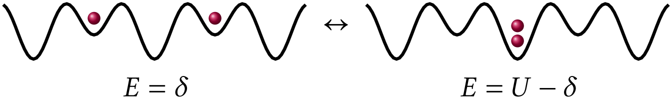


$$\hat{H}_{\mathrm{BHM}} = -J\sum_j\left(\hat{b}_j^\dagger\hat{b}_{j+1} + \mathrm{H.c.}\right) + \frac{U}{2}\sum_j\hat{n}_j\left(\hat{n}_j - 1\right) + \sum_j\left[(-1)^j\frac{\delta}{2} + j\gamma\right]\hat{n}_j.$$

Even sites = matter,   odd sites = gauge.

B. Yang et al., Nature **587**, 392 (2020).

Gauge-invariant hopping

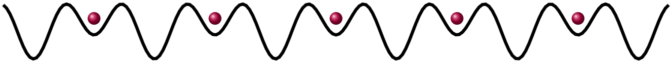


- Corresponds to $\hat{\psi}_i \hat{S}^+_{i,i+1} \hat{\psi}_{i+1}$.
- Need $\delta \approx U/2$.
- Tilt γ suppresses other processes.

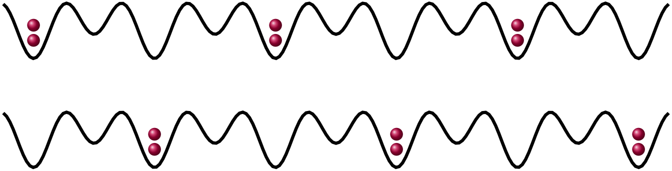
$$\kappa \approx \frac{4\sqrt{6}J^2}{U}, \qquad m \approx \delta - \frac{U}{2}.$$

Gauge-invariant ground states

$m \to -\infty$ (charge-proliferated):

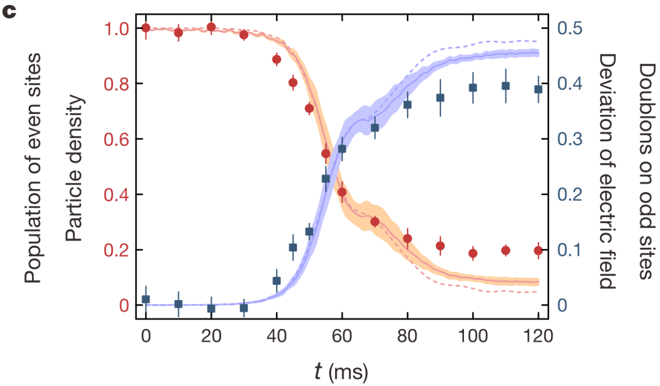


$m \to \infty$ (vacuum):



Experimental results

Ramp from $m = -\infty \to \infty$:

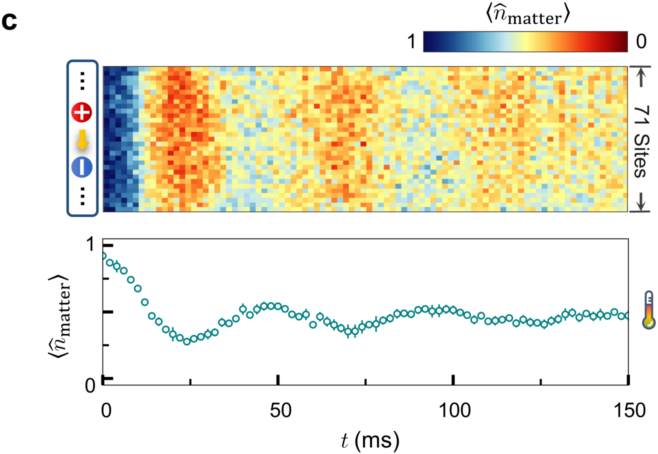


Data points = experiment, curves = numerics.

B. Yang et al., Nature **587**, 392 (2020).

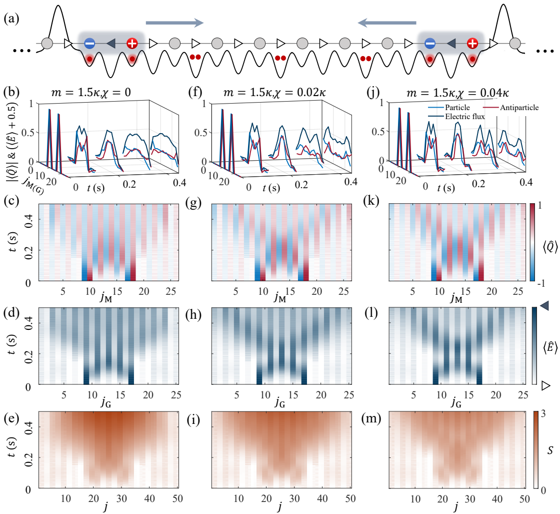
Global quench

Sudden quench $m = -\infty \to 0$:



Z.-Y. Zhou et al., Science **377**, 311 (2022).

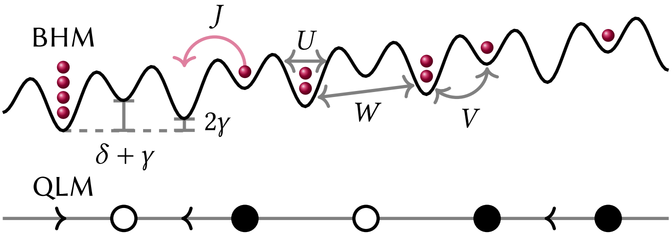
Particle collision (numerical proposal)



χ = background field (confining potential).

G.-X. Su, JO, J. C. Halimeh, arXiv:2401.05489 (2024).

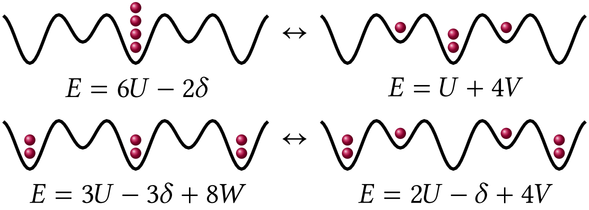
Upgrading to spin-1



$$\hat{H}_{\mathrm{BHM}} = -J\sum_j\left(\hat{b}_j^\dagger\hat{b}_{j+1}+\mathrm{H.c.}\right)+\frac{U}{2}\sum_j\hat{n}_j\left(\hat{n}_j-1\right)$$
$$+\sum_j\left[(-1)^j\frac{\delta}{2}+j\gamma\right]\hat{n}_j+V\sum_j\hat{n}_j\hat{n}_{j+1}+W\sum_{j\text{ odd}}\hat{n}_j\hat{n}_{j+2}.$$

JO, B. Yang, I. P. McCulloch, P. Hauke, J. C. Halimeh, arXiv:2305.06368 (2023).

Gauge-invariant hopping (spin-1)



$E = 6U - 2\delta \leftrightarrow E = U + 4V$

$E = 3U - 3\delta + 8W \leftrightarrow E = 2U - \delta + 4V$

2 constraints:

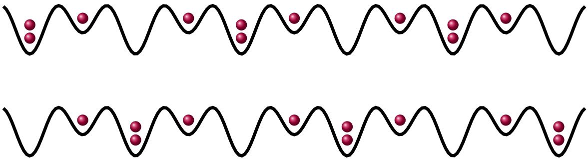
$$5U - 2\delta - 4V \approx 0, \qquad U - 2\delta - 4V + 8W \approx 0.$$

Using $V = 2W$ to avoid some unwanted resonances, we obtain

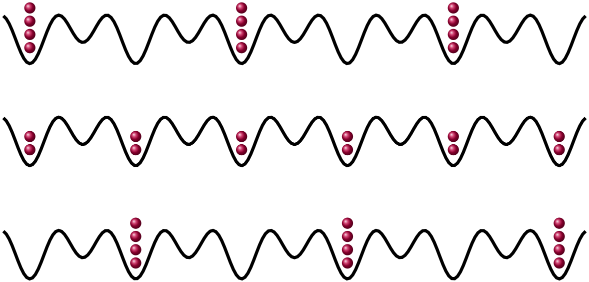
$$\hat{E} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad \hat{U} = \frac{1}{\sqrt{6}} \begin{pmatrix} 0 & 0 & 0 \\ \sqrt{12} & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{pmatrix}, \quad \kappa = \frac{16\sqrt{6} J^2 (2\delta - 3U)}{(2\delta - 3U)^2 - 16\gamma^2}, \quad \mu = -\frac{3}{2}U + \delta + 2V - 2W + \frac{16 J^2 (5U - 6\delta)}{(5U - 6\delta)^2 - 16\gamma^2}, \quad g^2 = 4U - 8W.$$

Gauge-invariant ground states

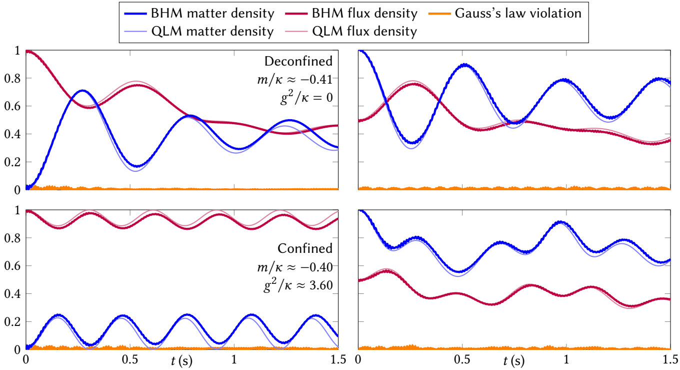
$m \to -\infty$ (charge-proliferated):



$m \to \infty$ (vacuum):



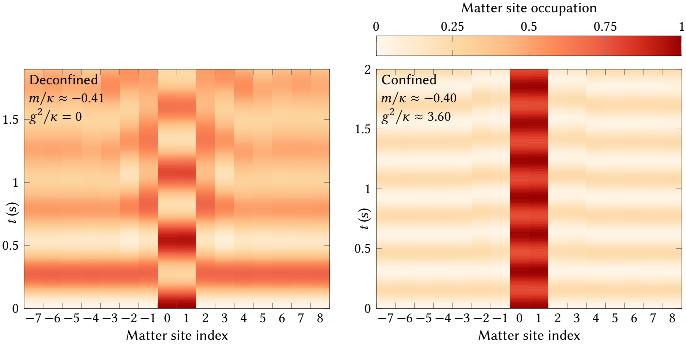
Global quenches (numerics)



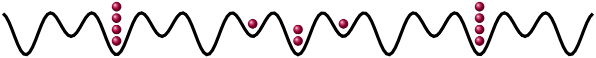
Initial states:

JO, B. Yang, I. P. McCulloch, P. Hauke, J. C. Halimeh, arXiv:2305.06368 (2023).

Pair confinement (numerics)



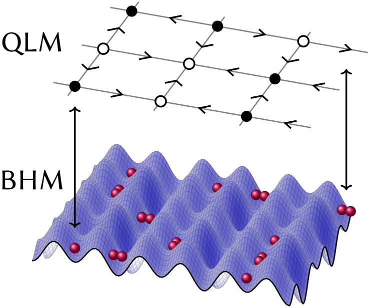
Initial state:



JO, B. Yang, I. P. McCulloch, P. Hauke, J. C. Halimeh, arXiv:2305.06368 (2023).

Extending to 2+1D

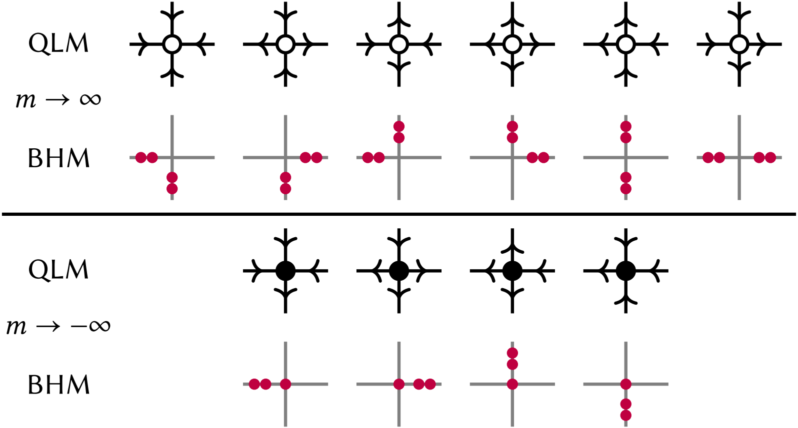
$$\begin{aligned}\hat{H}_{\mathrm{BHM}} =& -J\sum_{\langle i,j\rangle}\left(\hat{b}_i^\dagger\hat{b}_j+\mathrm{H.c.}\right)\\ &+\frac{U_j}{2}\sum_j \hat{n}_j\left(\hat{n}_j-1\right)\\ &+\sum_j\left[\vec{\gamma}\cdot j-\delta_j-\eta_j\right]\hat{n}_j.\end{aligned}$$



- $U_j = \alpha U$ on matter sites, U elsewhere.
- $\delta_j = \delta$ on gauge sites, 0 elsewhere.
- $\eta_j = \eta$ on ‘forbidden’ sites, 0 elsewhere.
- Two different tilts for each axis $\vec{\gamma} = (\gamma_x, \gamma_y)$.
- Hardcore bosonic matter.

JO, I. P. McCulloch, B. Yang, P. Hauke, J. C. Halimeh, arXiv:2211.01380 (2022).

Gauge-invariant configurations



QLM

$m \to \infty$

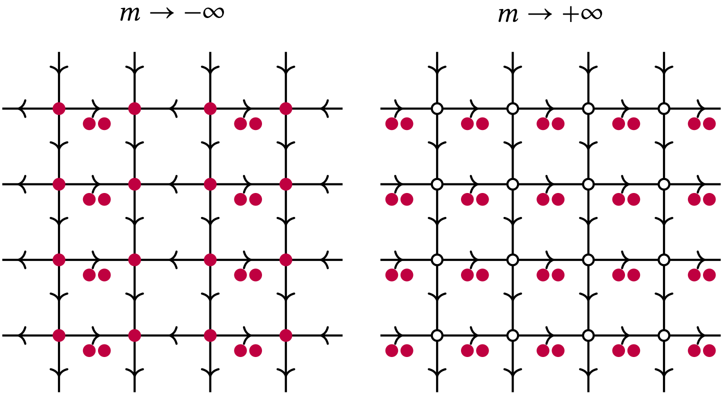
BHM

QLM

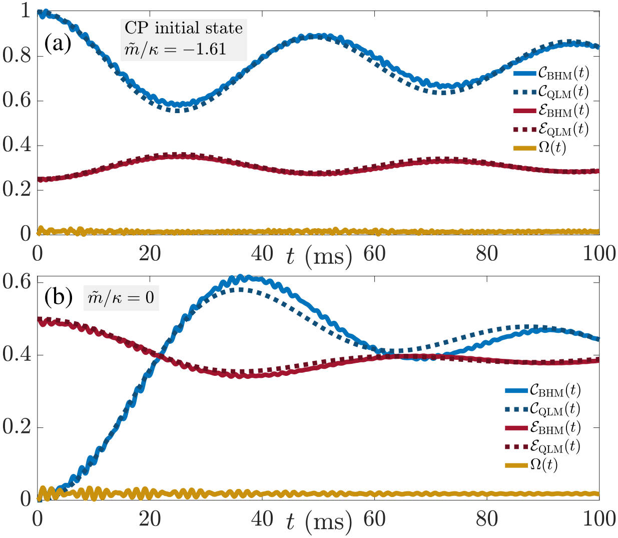
$m \to -\infty$

BHM

Gauge-invariant configurations



Global quenches (numerics)



JO, I. P. McCulloch, B. Yang, P. Hauke, J. C. Halimeh, arXiv:2211.01380 (2022).

Conclusion

Thanks to collaborators:

- Ian McCulloch (NTHU, formerly UQ).
- Jad Halimeh (LMU).
- Philipp Hauke (U. Trento).
- Bing Yang (SUSTech).
- Guoxian Su (Heidelberg U.)